IDENTIFICATION OF FIRST-ORDER PLUS DEAD-TIME CONTINUOUS-TIME MODELS USING TIME-FREQUENCY INFORMATION

George Acioli Júnior*, Marcus A. R. Berger*, Isabela S. Vieira*, Péricles R. Barros*

* *Unidade Acadêmica de Eng. Elétrica*
Univ. Fed. de Campina Grande
Campina Grande, Paraíba, Brazil

Emails: georgeacioli@yahoo.com.br, marcusarb@yahoo.co.uk, isabela.svieira@bol.com.br, prbarros@dee.ufcg.edu.br

Abstract— In this work the identification of first-order plus dead-time models from time and frequency domain experiments is considered. Alternative techniques for identification are examined and simple algorithms are proposed for dealing with the dead-time. Simulation examples are used to illustrate the techniques.

Keywords— Continuous-time identification; Process identification; Time delay process; Instrumental variables.

Resumo— Neste trabalho a identificação de modelos de primeira ordem com atraso através de experimentos no domínio do tempo e da frequência é considerada. Técnicas alternativas de identificação são examinadas e algoritmos simples que tratam do atraso são propostos. Exemplos de simulação são usados para ilustrar as técnicas.

Keywords— Identificação no Tempo Contínuo; Identificação de Processos; Processos com Tempo de Atraso; Variáveis Instrumentais.

1 Introduction

The estimation of continuous-time models from sampled data has received some attention in the last years, motivated by the need of such models to recover physical parameters or to allow the use of design techniques developed for continuous-time controllers. An extensive list of references on the subject can be found in (Mensler, 1999), in which a detailed survey discusses the advantages of a direct approach in relation to the indirect estimation of a discrete-time model plus a later transformation into a continuous-time model. Several papers have been presented in recent conferences (for instance, 13th IFAC Symposium on System Identification (SYSID 2003) and 16th IFAC World Congress 2005) to report new developments and applications.

The continuous-time results reported in the literature mainly address finite-dimensional systems. But dead-time is present in several industrial processes so that simple models such as first and second order dead-time continuous time one are widely used to tune industrial controllers. In the design of PID controllers the process model that receives most attention is first-order plus dead-time model (FOPDT) (Sudaresan and Krishnaswamy, 1977). There are a few methods to estimate parameters for this model. Among them one can mention the graphics and the area methods (Åström and Hägglund, 1995). A method less sensitive to noise is proposed in (Wang et al., 1999) which uses least-squares method to estimate the parameters of FOPDT model. Variants of this methods are used in (Wang and Zhang, 2001) and (Wang et al., 2000). For such simple models the results are remarkably good and motivated the present work. Closed loop methods exist such the one presented in ((G. Acioli Júnior and Barros, 2006)).

In this paper three techniques are compared for the estimation of continuous-time systems from discrete-time measurements. The first technique is the one presented in (Coelho and Barros, 2003)) and requires only a step input. The second approach, starts with the use of an integrator relay test to obtain the transfer function response at the frequency for which the process has 90^o phase shift and uses excitation with twice this frequency to obtain another point of the process frequency response. The combination with the step input is used in a mixed time-frequency domain estimation problem to estimate the process with time delay. Finally, in the third case, the process is excited with a signal obtained by switching between two relay tests. The step response is recovered solving a constrained least-squares minimization which uses the Frequency Sampling Filters ((Wang and Garnier, 2005)). The FOPDT model is estimated applying the algorithm of the first technique to this step response.

This paper is organized as follows. In Section 2, the problem statement is presented. The continuous-time identification of FOPDT techniques are presented in in Section 3. In Section 4 the techniques are compared using simulations of examples and, finally, conclusions are presented in Section 5.

2 The Problem Statement

In this paper it is considered the identification of FOPDT continuous-time models represented by

$$G(s) = \frac{b}{s+a} e^{-Ls}. \qquad (1)$$

It is assumed open-loop operation and that the excitation applied to the process is restricted to a step response, simple relay tests or simple excitation generated from a relay-based signal. Although it is desired to estimate a continuous-time model, the available data to the estimation is discrete-time. The aim of the paper is to evaluate the improvements obtained, for such simple models, by the increase of the excitation content and the complexity of the techniques.

The excitation content is increased as follows. The simplest excitation is a step input. The excitation content is increased by using two relay tests (standard and integrator) as presented in (Åström and Hägglund, 1995). In the standard test a relay is applied in an unity feedback loop so that an oscillation develops at a frequency for which the process has phase 180^o. The integrator relay test has an additional integrator so that the oscillation develops at a frequency for which the process has phase 90^o. The relays tests are used in two ways. In first case, the integrator relay test is performed and the oscillation frequency $\hat{\omega}_g$ is measured. Then a square wave with frequency $2\hat{\omega}_g$ is also applied to the process. In the second case, the two relays are switched to generate a more exciting signal.

3 Techniques for the Identification of FOPDT Models

In this Section the identification techniques are described.

3.1 Technique 1: Identification of FOPDT Model from a Step Input

The process is assumed to be at steady-state at $t = 0$ so that, without loss of generality, $u(t) = 0$ for $t < 0$ and zero initial conditions are assumed. For $t \geq L$ the process is assumed to satisfy the differential equation

$$\dot{y}(t) + ay(t) = bu(t-L), \qquad (2)$$

where the disturbance term has been omitted.

Integrating Eq.(2) from $\tau = 0$ to $\tau = t$ yields

$$y(t) = -a\int_0^t y(\tau)\,d\tau + b\int_0^t u(\tau - L)\,d\tau.$$

In continuous-time identification, techniques which use such models (with $L = 0$) are named *Integral Methods* (Mensler, 1999). An integral method has been used in (Wang et al., 2000), with the process in open loop and under a step input with amplitude h applied at $t = 0$. For this case the model also satisfy

$$y(t) = -a\int_0^t y(\tau)\,d\tau + bht - bhL,$$

from which a regression model can be obtained on model parameters $\{a, b, bL\}$, from which L can be extracted. The estimate is computed using instrumental variable method (see (Ljung, 1999)).

Define

$$\begin{aligned}\phi(t) &= \begin{bmatrix} -\int_0^t y(\tau)\,d\tau & ht & -h \end{bmatrix}^T, \\ \hat{\theta} &= \begin{bmatrix} a & b & bL \end{bmatrix}^T, \\ \psi(t) &= \begin{bmatrix} -\int_0^t x(\tau)\,d\tau & ht & -h \end{bmatrix}^T. \end{aligned} \qquad (3)$$

Where $x(t) = \widehat{G}_{LS1}(t)u(t)$. $\widehat{G}_{LS1}(t)$ is the result of a direct least-squares estimation method. $\psi(t)$ is the instruments. For the instrumental variable method, form matrices

$$\begin{aligned} Y &= \begin{bmatrix} y(0) & y(T) & \cdots & y((N-1)T) \end{bmatrix}^T, \\ \Phi &= \begin{bmatrix} \phi(0) & \phi(T) & \cdots & \phi(((N-1)T)) \end{bmatrix}^T, \\ \Psi &= \begin{bmatrix} \psi(0) & \psi(T) & \cdots & \psi(((N-1)T)) \end{bmatrix}^T. \end{aligned}$$

Finally, compute the instrumental variable estimate

$$\hat{\theta}_{IV1} = \left(\Psi^T \Phi\right)^{-1} \Psi^T Y \qquad (4)$$

and the corresponding model $\hat{G}_{IV1}(s)$.

3.2 Technique 2: Identification of FOPDT Model from a Mixed Time-Frequency Least Squares

In the second technique, a mixed time-frequency domain estimation problem is used. A step response test is used to obtain the signals for the time part and the model from the previous technique is used. An integrator relay test is used to obtain, via DFT, the frequency response at the 90^o phase shift frequency $\hat{\omega}_g$. In addition, a square wave with twice this frequency ($2\hat{\omega}_g$) is used to obtain the second frequency point.

The mixed time-frequency minimization problem is a weighted estimation problem combining the step response test (with the regression model from the previous technique) with the two estimated frequency response points. For the frequency domain part, consider the approximation for model 1 given by $G(s) = \frac{b(1-sL)}{s+a}$. And that the frequency response at $G(j\omega) = \frac{b(1-j\omega L)}{j\omega + a}$ with $\omega \in \{\hat{\omega}_g, 2\hat{\omega}_g\}$.

For the same frequencies estimates $\hat{G}(j\omega)$ are available from the DFT on the composed excitation test. Thus, the following regression model can be obtained:

$$\hat{z} = x^T(\omega)\hat{\theta}$$

with

$$\hat{z} = j\omega\hat{G}(j\omega); \quad x^T(j\omega) = \left[-\hat{G}(j\omega) \quad 1 \quad -j\omega\right].$$

Now, define the cost function

$$J = \left(\frac{(1-\alpha)}{N}\sum_{i=1}^{N}(y-\hat{y})^2 + \frac{\alpha}{M}\sum_{j=1}^{M}(z-\hat{z})^2\right)$$

where N is the number of time samples and M is the number of frequencies points ($M = 2$) and α a constant weight in the range $[0,1]$. It should be noticed that the frequency information enters in the problem as part of the cost function. Rewrite the cost function in compact form by defining $\hat{Y} = \Phi\hat{\theta}$ and $\hat{Z} = X\hat{\theta}$, so that the cost function becomes

$$J = \frac{(1-\alpha)}{N}\left(Y - \Phi\hat{\theta}\right)^T\left(Y - \Phi\hat{\theta}\right) \tag{5}$$

$$+\frac{\alpha}{M}\left(Z - X\hat{\theta}\right)^T\left(Z - X\hat{\theta}\right). \tag{6}$$

Lemma: The solution $\hat{\theta}$ which minimizes the cost function 6 is given by

$$\hat{\theta}_{LS2} = \left[\frac{(1-\alpha)}{N}\Phi^T\Phi + \frac{\alpha}{M}X^TX\right]^{-1} \left[\frac{(1-\alpha)}{N}\Phi^TY + \frac{\alpha}{M}X^TZ\right].$$

Proof:

$$\frac{\partial J}{\partial\theta} = \frac{(1-\alpha)}{N}\left[-\left(Y^T\Phi\right)^T - \Phi^TY + \left(\Phi^T\Phi + \Phi^T\Phi\right)\hat{\theta}\right]$$
$$\frac{\alpha}{M}\left[-\left(Z^TX\right)^T - X^TZ + \left(X^TX + X^TX\right)\hat{\theta}\right] = 0.$$

Perform a few manipulations to obtain

$$\frac{(1-\alpha)}{N}\Phi^TY + \frac{\alpha}{M}X^TZ = \left[\frac{(1-\alpha)}{N}\Phi^T\Phi + \frac{\alpha}{M}X^TX\right]\hat{\theta}$$

from which the result follows if matrix $\left[\Phi^T\Phi + X^TX\right]$ is nonsingular.

Transfer function $\hat{G}_{LS2}(s)$ is directly obtained from $\hat{\theta}_{LS2}$. $\psi(t)$ is defined as in 3 and the instrumental variable estimate is given by

$$\hat{\theta}_{IV2} = \left[\frac{(1-\alpha)}{N}\Psi^T\Phi + \frac{\alpha}{M}X^TX\right]^{-1} \left[\frac{(1-\alpha)}{N}\Psi^TY + \frac{\alpha}{M}X^TZ\right].$$

Transfer function $\hat{G}_{IV2}(s)$ is directly obtained from $\hat{\theta}_{IV2}$.

3.3 Technique 3: Identification of FOPDT Model from Constrained Identification

In the third technique, the process step response is recovered using the procedure presented in ((Wang and Garnier, 2005)). The procedure solves a least-squares problem for the Frequency Sampling Filter coefficients plus constraints on time and frequency. In this paper only equality constraints are used. The first constraint is the assumption that the estimated 90^o frequency response point is true. The second constraint is that the step response is known to be zero up to a lower bound estimate for the time delay. The step response is obtained using an excitation obtained by switching between the two relay tests after a number of oscillation periods. A FOPDT model is estimated from the estimated step response using the least-squares algorithm of the first technique. It should be noticed that unlike the previous technique, here the frequency information enters in the problem as a restriction, as well as a-priori information on the time delay.

The general idea is as follows (for details see ((Wang and Garnier, 2005))): The step response is estimated using the discrete transfer function of the system which can be represented in terms of the frequency response coefficients via the frequency sampling filters(FSF) expression:

$$G(z) = \sum_{l=-\frac{n-1}{2}}^{\frac{n-1}{2}} G(e^{jl\Omega})H^l(z) \tag{7}$$

where n is an odd number to represent the number of significant parameters in the FSF model and $H^l(z)$ are the frequency sampling filters. The estimated parameters are

$$\hat{\theta} = \begin{bmatrix} G(e^{j0}) \\ \mathrm{Re}(G(e^{j\Omega}) \\ \mathrm{Im}(G(e^{j\Omega}) \\ \vdots \\ \mathrm{Re}(G(e^{j\Omega\frac{n-1}{2}}) \\ \mathrm{Im}(G(e^{jl\Omega\frac{n-1}{2}}) \end{bmatrix}$$

The step response of the system at the sample m has a linear relation to $\hat{\theta}$ via

$$g_m = Q(m)^T\hat{\theta} \tag{8}$$

where

$$Q(m) = \begin{bmatrix} \frac{m+1}{N} \\ 2\,\mathrm{Re}(S(1,m)) \\ 2\,\mathrm{Im}(S(1,m)) \\ \vdots \\ 2\,\mathrm{Re}(S(\frac{n-1}{2},m)) \\ 2\,\mathrm{Im}(S(\frac{n-1}{2},m)) \end{bmatrix}$$

$$S(l,m) = \frac{1}{N}\frac{1-e^{jl\Omega(m+1)}}{1-e^{j\Omega}}, l = 1,2,...,\frac{n-1}{2}.$$

Equality constraints in the time and frequency domains can be expressed in a linear form

$$M\theta = \gamma.$$

In this paper the frequency information is the 90^o frequency response point and a lower bound on the time delay. By defining

$$E = \sum_{k=1}^{M} \left[\phi_D(k)\phi_D(k)^T\right]$$
$$F = -2\sum_{k=1}^{M} \left[\phi_D(k)y_D(k)\right],$$

where $\phi_D(k)$ and $y_D(k)$ are the (FSF) regressor and output signals, respectively (see (Wang and Garnier, 2005)), the optimal least-squares with equality constraints solution has a closed-form as

$$\begin{bmatrix} E & M^T \\ M & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} -F \\ \gamma \end{bmatrix}$$

which can be solved explicitly as

$$\lambda_1 = -(ME^{-1}M^T)^{-1}(\gamma + ME^{-1}F)$$
$$\hat{\theta} = -E^{-1}(F + M^T\lambda_1)$$

and the corresponding model $\hat{G}_{LS3}(s)$ is estimated using the least-squares algorithm of the first technique and the step response estimated by 8.

4 Simulation Examples

In this section the identification techniques are applied to three processes. The cost function used to compare the estimates is

$$\varepsilon = \frac{1}{N}\sum_{k=0}^{N-1} \left[x(kT) - \hat{x}(kT)\right]^2$$

where $x(kT_s)$ is the actual process output (without noise), while $\hat{x}(kT_s)$ is the output of the simulation of the estimated process under the same step setpoint. In the first simulation there is no noise while in the second one a normally distributed noise with zero mean and variance 0.2 is added to the output. In all simulations $\alpha = 0.2$ for the second technique.

4.1 Example 1

In the first example it is used a FOPDT model

$$G_1(s) = \frac{1}{s+1}e^{-0.5s}.$$

4.1.1 No Noise Case

For the no noise case the estimates are

$$G_{IV1}(s) = \frac{1.01}{s+1.01}e^{-0.5216s}$$
$$G_{IV2}(s) = \frac{0.7952}{s+0.7699}e^{-0.5078s}$$
$$G_{LS3}(s) = \frac{1}{s+1}e^{-0.517s}.$$

The mean squared errors are

$$\varepsilon_1 = 7.2289\times10^{-6}, \varepsilon_2 = 0.0013, \varepsilon_3 = 6.9703\times10^{-6}.$$

In this case, the FSF constrained technique yields the best estimate in the mean squares sense.

4.1.2 Noisy Case

For the noisy case the estimates are

$$G_{IV1}(s) = \frac{0.7801}{s+0.7558}e^{-0.3907s}$$
$$G_{IV2}(s) = \frac{0.7610}{s+0.7254}e^{-0.4771s}$$
$$G_{LS3}(s) = \frac{1.117}{s+1.106}e^{-0.577s}.$$

The mean squared errors are

$$\varepsilon_1 = 9.4139\times10^{-4}, \varepsilon_2 = 0.0022, \varepsilon_3 = 2.0071\times10^{-4}.$$

Again, the FSF constrained technique yields the best estimate in the mean squares sense.

The model frequency responses are
$G_1(j\hat{\omega}_g) = 0.6559\measuredangle - 1.4308$
$G_1(j2\hat{\omega}_g) = 0.3985\measuredangle - 2.3117.$

The frequency responses errors are show in Table 1, so that the FSF constrained technique yields the best estimate at the frequency points. Simulation curves and Nyquist plot are shown in Figure 1 and 2.

Table 1: Frequency Responses Errors

$\hat{\omega}$	E_1	E_2	E_3
$\hat{\omega}_g$	8.93×10^{-2}	9.65×10^{-2}	4.39×10^{-2}
$2\hat{\omega}_g$	7.65×10^{-2}	8.32×10^{-2}	3.89×10^{-2}

where
$E_1 = |G_1(j\omega) - G_{IV1}(j\omega)|,$
$E_2 = |G_1(j\omega) - G_{IV2}(j\omega)|$
$E_3 = |G_1(j\omega) - G_{LS3}(j\omega)|.$

4.2 Example 2

The process is now given by

$$G_2(s) = \frac{4}{(s+1)(s+4)}e^{-0.5s}.$$

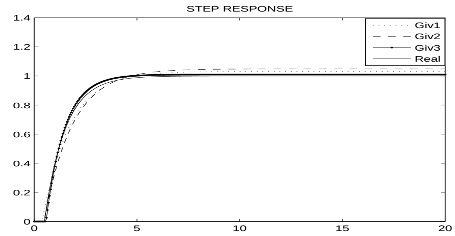

Figure 1: Step response for process 1.

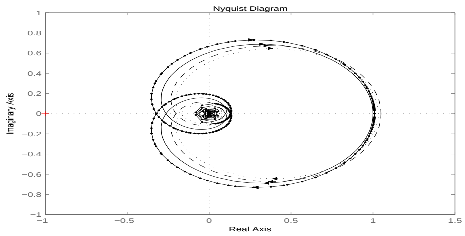

Figure 2: Nyquist plot for process 1

4.2.1 No Noise Case

For the no noise case the estimates are

$$G_{IV1}(s) = \frac{0.9417}{s+0.9416}e^{-0.7025s}$$
$$G_{IV2}(s) = \frac{0.7498}{s+0.7430}e^{-0.7050s}$$
$$G_{LS3}(s) = \frac{0.9566}{s+0.9548}e^{-0.728s}.$$

The mean squared errors are

$\varepsilon_1 = 2.80\times10^{-5}, \varepsilon_2 = 7.48\times10^{-4}, \varepsilon_3 = 3.63\times10^{-5}$.

Now, the first technique yields the best estimate in the mean squares sense.

4.2.2 Noisy Case

For the noisy case the estimates are

$$G_{IV1}(s) = \frac{0.7157}{s+0.7047}e^{-0.44135s}$$
$$G_{IV2}(s) = \frac{0.7682}{s+0.7503}e^{-0.7566s}$$
$$G_{LS3}(s) = \frac{0.9628}{s+0.975}e^{-0.746s}.$$

The mean squared errors are

$\varepsilon_1 = 7.4998\times10^{-4}, \varepsilon_2 = 0.0011, \varepsilon_3 = 1.9345\times10^{-4}$.

In this case, the FSF constrained technique yields the best estimate in the mean squares sense.

The model frequency responses are
$G_2(j\hat{\omega}_g) = 0.7193\measuredangle - 1.4229$
$G_2(j2\hat{\omega}_g) = 0.4362\measuredangle - 2.4140$.

The frequency responses errors are show in Table 2, so that again the FSF constrained technique yields the best estimate at the frequency points. Simulation curves and Nyquist plot are shown in Figure 3 and 4.

Table 2: Frequency Responses Errors

$\hat{\omega}$	E_1	E_2	E_3
$\hat{\omega}_g$	9.94×10^{-2}	6.99×10^{-2}	9.15×10^{-4}
$2\hat{\omega}_g$	7.12×10^{-2}	4.96×10^{-2}	2.83×10^{-2}

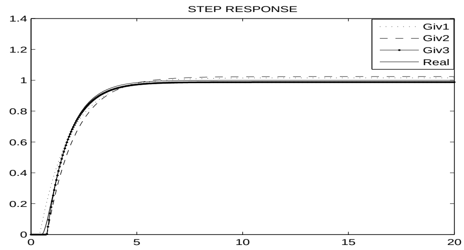

Figure 3: Step response for process 2.

4.3 Example 3

The process is now given by $G_3(s) = \frac{1}{(s+1)^4}$.

4.3.1 No Noise Case

For the no noise case the estimates are

$$G_{IV1}(s) = \frac{0.3526}{s+0.3511}e^{-1.2992s}$$
$$G_{IV2}(s) = \frac{0.4033}{s+0.3994}e^{-1.8096s}$$
$$G_{LS3}(s) = \frac{0.3053}{s+0.2861}e^{-1.25s}.$$

The mean squared errors are

$\varepsilon_1 = 0.0014, \quad \varepsilon_2 = 5.7126\times10^{-4}, \quad \varepsilon_3 = 0.0020.$

In this case, the mixed time-frequency technique yields the best estimate in the mean squares sense.

4.3.2 Noisy Case

For the noisy case the estimates are

$$G_{IV1}(s) = \frac{0.3659}{s+0.3593}e^{-1.3419s}$$
$$G_{IV2}(s) = \frac{0.4105}{s+0.4008}e^{-1.8030s}$$
$$G_{LS3}(s) = \frac{0.2916}{s+0.2713}e^{-1.19s}.$$

The mean squared errors are

$\varepsilon_1 = 0.0012, \quad \varepsilon_2 = 6.0253\times10^{-4}, \quad \varepsilon_3 = 0.0023.$

Again, the mixed time-frequency technique also yields the best estimate in the mean squares sense.

The model frequency responses are
$G_3(j\hat{\omega}_g) = 0.7509\measuredangle - 1.4959$
$G_3(j2\hat{\omega}_g) = 0.3829\measuredangle - 2.6619$.

The frequency responses errors are show in Table 3, so that the mixed time-frequency technique also yields the best estimate at the frequency points. Simulation curves and Nyquist plot are shown in Figure 5 and 6.

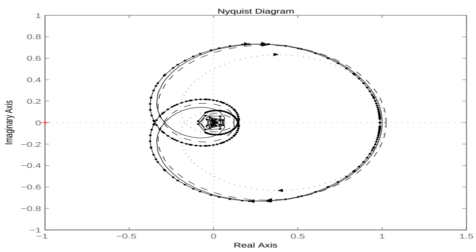

Figure 4: Nyquist plot for process 2.

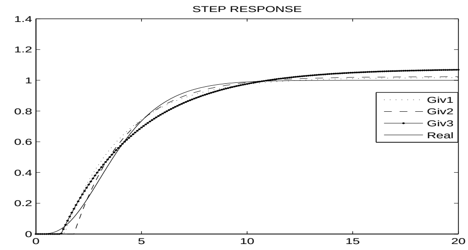

Figure 5: Step response for process 3.

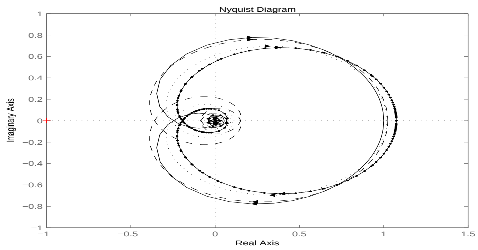

Figure 6: Nyquist plot for process 3.

Table 3: Frequency Responses Errors

$\hat{\omega}$	E_1	E_2	E_3
$\hat{\omega}_g$	6.33×10^{-2}	1.92×10^{-2}	0.14
$2\hat{\omega}_g$	4.10×10^{-2}	8.28×10^{-2}	3.19×10^{-2}

5 Conclusions

In this paper three techniques for the identification of continuous-time FOPDT models from discrete-time signals. The first technique uses a step response experiment to estimation. Two of the techniques use time and frequency domain information. The second one uses the information on a mixed time-frequency minimization problem while the third one uses the time information in the cost function and the frequency information plus an estimate of the time delay as equality constraints. Simulation results show the use of frequency data improved the estimation. The third technique yields better results when good frequency data is available. The step response mean square error could be used to choose between the three techniques.

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